Resonant Fibonacci quantum well structures in one dimension

A. N. Poddubny,[1,*] L. Pilozzi,[2] M. M. Voronov,[1] and E. L. Ivchenko[1]
[1]*A.F. Ioffe Physico-Technical Institute, 194021 St. Petersburg, Russia*
[2]*Istituto dei Sistemi Complessi, CNR, Casella Postale 10, Monterotondo Stazione, Rome I-00016, Italy*
(Received 23 November 2007; published 12 March 2008)

We propose a resonant one-dimensional quasicrystal, namely, a multiple quantum well (MQW) structure satisfying the Fibonacci-chain rule with the golden ratio between the long and short interwell distances. The resonant Bragg condition is generalized from the periodic to Fibonacci MQWs. A dispersion equation for exciton polaritons is derived in the two-wave approximation; the effective allowed and forbidden bands are found. The reflection spectra from the proposed structures are calculated as a function of the well number and detuning from the Bragg condition.

DOI: 10.1103/PhysRevB.77.113306 PACS number(s): 42.70.Qs, 61.44.Br, 71.35.−y

I. INTRODUCTION

The concept of quasicrystal as a nonperiodic structure with perfect long-ranged bond orientational order was brought in solid-state physics by Levine and Steinhardt.[1] It was extended to optics in Ref. 2, where a one-dimensional (1D) quasicrystal model constructed of dielectric layers forming the Fibonacci sequence was proposed. At just the same time, the concept of photonic crystals was suggested by Yablonovitch[3] and John.[4] Since then, the 1D photonic Fibonacci quasicrystals have been extensively studied.[5–8]

In this Brief Report, we introduce a nano-object, the Fibonacci quantum well (QW) structure, with interwell spacings arranged in the Fibonacci sequence. This means that the thickness of barriers separating the wells can take one of the two values so that the ratio between the long and short interwell spacings equals the golden mean $\tau=(\sqrt{5}+1)/2$. We focus on the light propagation in such a medium in the frequency region around the resonance frequency ω_0 of a two-dimensional exciton in the quantum well. The barriers are assumed to be thick enough so that the excitons in different wells are coupled only via electromagnetic field. Thus, the object under study is a resonant photonic quasicrystal, an intermediate structure between completely ordered and disordered media, namely, periodic multiple quantum wells (MQWs) with a fixed interwell spacing and MQWs with random interwell spacing.

Among periodic QW structures of particular interest are the resonant Bragg structures with the period satisfying the Bragg condition,

$$q(\omega_0)d = \pi j, \quad j = 1,2, \dots , \tag{1}$$

where $q(\omega)=\omega n_b/c$, $q(\omega_0)$ is the light wave vector at the exciton resonance frequency ω_0, n_b is the background refractive index of both QW and barrier materials, d is the structure period, and c is the light velocity. The periodic resonant Bragg MQWs have been first considered theoretically in Ref. 9 and then investigated in a number of theoretical as well as experimental works.[10–17] It was established that, for small enough numbers N of QWs (super-radiant regime), the optical reflection spectrum is described by a Lorentzian with the half width $N\Gamma_0+\Gamma$, where Γ_0 and Γ are, respectively, the exciton radiative and nonradiative damping rates in a single QW.[9,14] For a large number of wells (photonic crystal regime), the reflection coefficient is close to unity within the forbidden gap for exciton polaritons propagating in infinite periodic system and rapidly decreases near the gap edges $\omega_0-\Delta/\sqrt{j}$ and $\omega_0+\Delta/\sqrt{j}$, where $\Delta=\sqrt{2\omega_0\Gamma_0/\pi}$.[13–16]

In Sec. II, we will show that a generalized resonant Bragg condition analogous to Eq. (1) can be formulated for the resonant Fibonacci MQW structures, although the latter are aperiodic. In Sec. III, the significance of the proposed condition is verified by numerical calculations of the reflection spectra from the structures tuned on and slightly detuned from this condition, and the dependence of the reflection spectra on the number of wells is analyzed and compared with those for the periodic Bragg structures. In Sec. IV, we apply a two-wave approximation in order to determine the band gaps in the exciton-polariton spectrum of the Fibonacci structures and show that the simple analytic theory allows one to interpret quite well the numerical results.

II. RESONANT BRAGG CONDITION FOR FIBONACCI MULTIPLE QUANTUM WELL STRUCTURE

The structure under consideration is schematically depicted in Fig. 1. It consists of N identical QWs embedded in a matrix with dielectric constant ε_b. The interwell distances take two values represented by long and short segments of length l and s, respectively. For the Fibonacci chain, the coordinate z_m of the mth QW center is given by[18]

$$z_m = \bar{d}(m-1) + \frac{s}{\tau}\left(\frac{1}{\tau} - \left\{\frac{m}{\tau}\right\}\right), \tag{2}$$

where the integer m runs from 1 to N, τ is the golden ratio, $\bar{d}$ is the average period of the structure given by the product $s(3-\tau)$, and $\{x\}$ is the fractional part of x. An alternative way

FIG. 1. Scheme of the Fibonacci QW structure $\mathcal{F}_6$ with N=9 QWs.

 ©2008 The American Physical Society

of defining z_m is based on the recurrence relation $\mathcal{F}_{j+1} = \{\mathcal{F}_j, \mathcal{F}_{j-1}\}$ for finite Fibonacci chains of the order $j+1$, j, and $j-1$, with initial conditions of $\mathcal{F}_1 = S$ and $\mathcal{F}_2 = L$, where S and L are the segments with lengths s and $l = \tau s$, respectively.[18] Then, z_m are coordinates of boundaries between the segments in the $\mathcal{F}_j$ sequence.

The exact reflection coefficient of the light normally incident on such a structure from the left half-space can be obtained by standard transfer-matrix method.[19] In order to form the base for formulation of the resonant Bragg condition for the Fibonacci structures, we will analyze the reflection in the first-order Born approximation neglecting multireflection processes and summing up the amplitudes of waves reflected from distinct wells. Then, the amplitude reflection coefficient $r_N(\omega)$ from the N-well Fibonacci structure at the light frequency ω is given by

$$r_N(\omega) \approx N f[q(\omega), N] r_1(\omega), \qquad (3)$$

where $f(q,N)$ is the structure factor of the system,

$$f(q,N) = \frac{1}{N} \sum_{m=1}^{N} e^{2iqz_m},$$

and r_1 is the reflection coefficient from a single QW,

$$r_1(\omega) = \frac{i\Gamma_0}{\omega_0 - \omega - i(\Gamma + \Gamma_0)}.$$

For the semi-infinite Fibonacci MQWs, the structure factor $f(q) = \lim_{N\to\infty} f(q,N)$ can be presented in the following analytical form:[18]

$$f(q) = \sum_{h,h'=-\infty}^{\infty} \delta_{2q,G_{hh'}} f_{hh'}, \quad G_{hh'} = \frac{2\pi}{\bar{d}}(h + h'/\tau), \qquad (4)$$

$$f_{hh'} = \frac{\sin S_{hh'}}{S_{hh'}} \exp\left(i\frac{\tau - 2}{\tau} S_{hh'}\right), \quad S_{hh'} = \frac{\pi\tau}{\tau^2 + 1}(\tau h' - h). \qquad (5)$$

Allowed diffraction vectors $G_{hh'}$ form a dense pseudocontinuous set. The largest values of $|f_{hh'}|$ are reached for the pairs (h,h') coinciding with two successive Fibonacci numbers (F_j, F_{j-1}), with F_j defined recursively by $F_0 = 0$, $F_1 = 1$, and $F_{j+1} = F_j + F_{j-1}$. Thus, for $(h,h') = (F_j, F_{j-1}) = (1,0)$, $(1,1)$, $(2,1)$, $(3,2)$, and $(5,3)$ corresponding to $j = 1, \ldots, 5$, the modulus of $f_{hh'}$ equals ≈ 0.70. 0.88, 0.95, 0.98, and 0.99, respectively. For (h,h') not belonging to this particular set, values of $|f_{hh'}|$ are significantly smaller. As a consequence, the structure factor [Eq. (4)] reaches maxima when $2q(\omega)$ equals $G_{F_j,F_{j-1}}$. It follows then that if the exciton resonance frequency satisfies the condition

$$\frac{\omega_0 n_b}{c} \bar{d} = \pi\left(F_j + \frac{F_{j-1}}{\tau}\right), \quad j = 1, 2, \ldots, \qquad (6)$$

the coefficient [Eq. (3)] at $\omega = \omega_0$ and large N amounts to

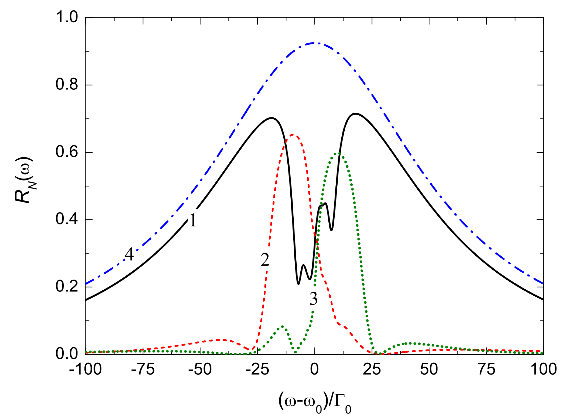

FIG. 2. (Color online) Reflection spectra calculated for three Fibonacci structures satisfying the resonant Bragg condition [Eq. (6)] for $j=2$ (curve 1) and detuned by $\pm 2\%$ from this condition (curves 2 and 3) in comparison with the reflection spectrum from the periodic resonant Bragg QW structure (curve 4). The values of parameters are indicated in text.

$$r_N = N f[q(\omega_0), N] r_1(\omega_0) \approx -\frac{N\Gamma_0 f_{hh'}}{\Gamma_0 + \Gamma},$$

which is of the same order of magnitude as the reflection coefficient calculated in the same Born approximation for a periodic resonant Bragg structure satisfying Eq. (1). Hence, Eq. (6) is indeed a resonant Bragg condition generalized for the Fibonacci MQWs. In the following, we fix the value of ω_0, consider the average period $\bar{d}$ as a variable parameter, and use the notation $\bar{d}_j$ for $\bar{d}$ given by Eq. (6) for the integer j. The corresponding thicknesses s_j, l_j of the short and long segments are related with $\bar{d}_j$ by

$$s_j = \bar{d}_j/(3 - \tau), \quad l_j = \bar{d}_j \tau/(3 - \tau). \qquad (7)$$

The estimation [Eq. (3)] for r_N is valid until $|r_N| \ll 1$, i.e., if $N\Gamma_0 \ll \max\{|\omega_0 - \omega|, \Gamma\}$. Otherwise, one has to take into account the multireflection of the light waves from QWs, which is readily achieved by the standard transfer-matrix numerical calculation. The results are presented and analyzed in the next section.

III. CALCULATED REFLECTION SPECTRA

Figure 2 presents reflection spectra calculated for four structures containing $N = 50$ quantum wells. The exciton parameters used are as follows: $\hbar\omega_0 = 1.533$ eV, $\hbar\Gamma_0 = 50$ μeV, $\hbar\Gamma = 100$ μeV, and $n_b = 3.55$. Curve 1 is calculated for the resonant Fibonacci QW structure satisfying the exact Bragg condition [Eq. (6)] with $j = 2$, so that $\bar{d} = \bar{d}_2$, $s = s_2$, and $l = l_2$. Curves 2 and 3 correspond to the Fibonacci structures with the barrier thicknesses slightly detuned from s_2 and l_2: $s/s_2 = l/l_2 = 1.02$ for curve 2 and $s/s_2 = l/l_2 = 0.98$ for curve 3. Curve 4 describes the reflection from the periodic Bragg structure with the same exciton parameters and the period $d = \pi/q(\omega_0)$, satisfying Eq. (1). From comparison of curves 1 and 4, we conclude that the reflection spectra from the reso-

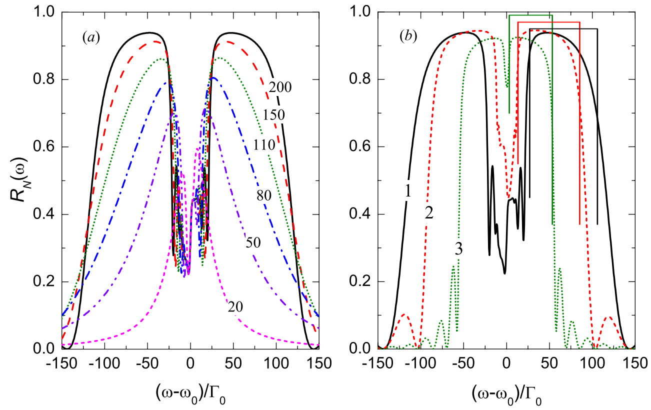

FIG. 3. (Color online) Reflection spectra from the resonant Fibonacci structures. (a) Six curves are calculated for the structures satisfying the condition [Eq. (6)] with $j=2$ and $N=20,50,80,110,150,200$. The number of wells is indicated near each corresponding curve. (b) Curves 1, 2, and 3 are calculated for the structures with $N=200$ and indices $j=2,3,5$ in Eq. (6). Vertical lines connected by a horizontal bar indicate the exciton-polariton high-frequency gap given by Eq. (10). Other parameters are the same as in Fig. 2.

nant periodic and Fibonacci structures tuned to the Bragg conditions [Eqs. (1) and (6)] are close to each other outside the frequency region around ω_0. Moreover, it follows from curves 2 and 3 that a slight deviation from the condition [Eq. (6)] results in a radical decrease of the effective spectral half width. Thus, the sensitivity to the resonance condition, the characteristic of periodic Bragg QW systems, holds also for aperiodic QW systems such as the Fibonacci structures. The remarkable structured dip in the middle of the spectrum 1 is the only qualitative difference from the periodic structures; the origin of this dip is explained in the next section. Now, we turn to the analysis of reflection spectra as a function of the QW number N and index j in Eq. (6).

Evolution of the reflection spectra with the QW number N is illustrated in Fig. 3(a). The spectral envelope smoothed to ignore dip in the middle shows a behavior similar to that of the conventional Bragg QW structure. Indeed, for small N, the envelope is a Lorentzian with the half width increasing as a linear function of N. This is a straightforward manifestation of super-radiant regime, which, as we can see here, does not necessarily require periodicity even if the interwell distances are comparable to the light wavelength. The saturation of the spectral half width (photonic crystal regime) begins at large N of the order of $\sqrt{\omega_0/\Gamma_0}$, in a similar way as for the periodic Bragg structures. The shape of the spectra for large N allows us to suppose existence of two wide symmetrical stop bands in the energy spectrum of the structure with an allowed band between them. Of course, the application of terms "allowed" and "stop" bands to an aperiodic structure is questionable. In Sec. IV, we show that, nevertheless, these terms are applicable in a reasonable approximation.

Figure 3(b) presents the reflection spectra of the Fibonacci QW structures containing a large number of wells, $N=200$, and satisfying Eq. (6) with three different values of j. All the curves indicate an existence of the stop and allowed bands. However, the bandwidths are j dependent: the stop band (or gap) indicated by a united pair of vertical lines and the middle dip are both squeezed with increase of j.

IV. EXCITON-POLARITON ENERGY SPECTRUM

For the light propagating in a system of identical QWs located at the points z_m $(m=1,2,\ldots)$, the equation for the electric field can be written as[19]

$$\left(-\frac{d^2}{dz^2}-q^2\right)E(z)=\frac{2q\Gamma_0}{\omega_0-\omega-i\Gamma}\sum_m \delta(z-z_m)E(z_m), \quad (8)$$

where $q\equiv q(\omega)$, and we assume that quantum wells are thin as compared to the light wavelength. We consider a Fibonacci multi-QW structure with large N and the average period $\bar{d}_j$ satisfying Eq. (6) for a certain value of j. The assumed large number of wells allows us to replace the structure factor $f(q,N)$ by $f(q)\equiv f(q,\infty)$. Using the properties of the coefficients $f_{hh'}$ in Eq. (4) for $f(q)$, we can retain in the sum [Eq. (4)] only one term $f_{hh'}\delta_{2q,G_{hh'}}$ with $(h,h')=(F_j,F_{j-1})$. In other words, we take into account only one diffraction vector $G_{hh'}$ corresponding to the condition [Eq. (6)] and neglect all other possible diffraction vectors. In this approximation, we can present the light wave inside the Fibonacci structure as a superposition of normal waves, each of them being a sum of two plane waves with the wave vectors K and $K'=K-G_{hh'}$ with $K\approx G_{hh'}/2$. The amplitudes of the chosen spatial harmonics, E_K and $E_{K'}$, satisfy the following two coupled equations:

$$(q^2-K^2+\chi)E_K+\chi f^*_{hh'}E_{K'}=0,$$

$$\chi f_{hh'}E_K+(q^2-K'^2+\chi)E_{K'}=0, \quad (9)$$

where

$$\chi=\frac{2q\Gamma_0}{\bar{d}(\omega_0-\omega-i\Gamma)}.$$

In the following analysis, we ignore the exciton dissipation, neglecting the nonradiative damping. Thus, the frequency axis is divided into intervals of purely allowed and forbidden bands with propagating and evanescent polaritonic solutions. In the allowed bands, the solutions are characterized by real values of the wave vector K. It is convenient to reduce the exciton-polariton dispersion $\omega(K)$ to the "first Brillouin zone" defined in the interval $-G_{hh'}/2<K\leqslant G_{hh'}/2$. The detailed behavior of $\omega(K)$ inside this interval lies out of the scope of the present Brief Report. Note that, in close vicinity to ω_0, the two-wave approximation is inadequate and the polariton dispersion should be calculated, taking into account an admixture of a lot of plane waves. Here, we consider only the exciton-polariton eigenfrequencies at the edge of the Brillouin zone, $K=-K'=G_{hh'}/2$, which identify stop bands and where the two-wave approximation is accurate. It follows from Eq. (9) that four eigenfrequencies at this point are given by

$$\omega^{\pm}_{\rm out}=\omega_0\pm\Delta\sqrt{\frac{1+|f_{hh'}|}{2(h+h'/\tau)}},$$

$$\omega_{\rm in}^{\pm} = \omega_0 \pm \Delta \sqrt{\frac{1-|f_{hh'}|}{2(h+h'/\tau)}}. \qquad (10)$$

In accordance with Fig. 3, we attribute the interval $\omega_{\rm in}^{+} < \omega < \omega_{\rm out}^{+}$ to the exciton-polariton upper stop band (labeled by index "+") and the interval between $\omega_{\rm out}^{-}$ and $\omega_{\rm in}^{-}$ to the lower stop band (labeled by index "−"). The subscripts "in" and "out" denote the stop-band edges, inner and outer, with respect to ω_0.

The values of $\omega_{\rm in}^{+}$ and $\omega_{\rm out}^{+}$ are marked by vertical lines in Fig. 3(b). One can see an excellent agreement between the band edges revealed in the calculated spectra and those given by Eq. (10) which unambiguously confirms the interpretation of the frequencies [Eq. (10)].

Equation (10) can be reduced to those for the periodic resonant Bragg structures as soon as $|f_{hh'}|$ is set to unity and $h+h'/\tau$ is replaced by the integer j. For $|f_{hh'}|=1$, the inner eigenfrequencies merge at ω_0 and a single band gap of width $2\Delta/\sqrt{j}$ is formed. In the Fibonacci QW structures, $|f_{hh'}|<1$ and, as a result, an allowed band opens between $\omega_{\rm in}^{-}$ and $\omega_{\rm in}^{+}$. We note that a qualitatively similar band structure can be realized when the periodic MQWs have a compound elementary cell.[15] One can easily show that, also in this case, the modulus of the structure factor is smaller than unity. Moreover, Eq. (10) can be reduced to Eq. (26) of Ref. 15 if $h+h'/\tau$ and $|f_{hh'}|$ are replaced, respectively, by $1/2$ and $|\cos qd_2|$, where d_2 is the interwell distance in the compound unit cell of a periodic structure with two QWs in the supercell.

In the Fibonacci QW structure, the decrease of stop-band widths with the increasing $h+h'/\tau$ is related to the corresponding increase of the average period $\bar{d}$ in Eq. (6), and it is analogous to the $j^{-1/2}$ power law of the bandwidth for the periodic resonant Bragg structures. The middle allowed bandwidth decreases even faster because, as mentioned above, the value of $|f_{hh'}|$ tends to unity and, therefore, the value of $\sqrt{1-|f_{hh'}|}$ rapidly vanishes as the index j in Eq. (6) changes from 2 to 5.

V. CONCLUSIONS

We have introduced into consideration resonant 1D photonic quasicrystals based on Fibonacci QW structures. The analysis of light reflection in the Born approximation has been used to formulate the resonant Bragg condition for this system. The results of straightforward transfer-matrix numerical calculation confirm the relevance of the generalized Bragg condition imposed on the aperiodic system under study. For a small number N of QWs, the Fibonacci structures show the super-radiant behavior, while for high values of N exceeding $\sqrt{\omega_0/\Gamma_0}$, the photonic crystal regime with distinct stop bands in optical spectra is reached. A qualitative difference with respect to the periodic resonant Bragg QW structures lies in the presence of a structured dip in the reflection spectrum around the exciton resonance frequency ω_0. An approximate two-wave exciton-polariton model allows one to describe the widths of the allowed and forbidden bands as a function of the structure parameters.

ACKNOWLEDGMENTS

This work was supported by RFBR and the "Dynasty" Foundation—ICFPM.

*poddubny@coherent.ioffe.ru

[1] D. Levine and P. J. Steinhardt, Phys. Rev. Lett. **53**, 2477 (1984).
[2] M. Kohmoto, B. Sutherland, and K. Iguchi, Phys. Rev. Lett. **58**, 2436 (1987).
[3] E. Yablonovitch, Phys. Rev. Lett. **58**, 2059 (1987).
[4] S. John, Phys. Rev. Lett. **58**, 2486 (1987).
[5] T. Hattori, N. Tsurumachi, S. Kawato, and H. Nakatsuka, Phys. Rev. B **50**, 4220 (1994).
[6] M. A. Kaliteevski, V. V. Nikolaev, R. A. Abram, and S. Brand, Opt. Spectrosc. **91**, 109 (2001).
[7] R. W. Peng *et al.*, Appl. Phys. Lett. **80**, 3063 (2002).
[8] S. V. Zhukovsky and A. V. Lavrinenko, Photonics Nanostruct. Fundam. Appl. **3**, 129 (2005).
[9] E. L. Ivchenko, A. I. Nesvizhskii, and S. Jorda, Fiz. Tverd. Tela (Leningrad) **36**, 2118 (1994) [Phys. Solid State **36**, 1156 (1994)].
[10] V. P. Kochereshko, G. R. Pozina, E. L. Ivchenko, D. R. Yakovlev, A. Waag, W. Ossau, G. Landwehr, R. Hellmann, and E. O. Göbel, Superlattices Microstruct. **15**, 471 (1994).
[11] J. Sadowski, H. Mariette, A. Wasiela, R. André, Y. M. d'Aubigné, and T. Dietl, Phys. Rev. B **56**, R1664 (1997).
[12] J. P. Prineas, C. Ell, E. S. Lee, G. Khitrova, H. M. Gibbs, and S. W. Koch, Phys. Rev. B **61**, 13863 (2000).
[13] E. L. Ivchenko and M. Willander, Phys. Status Solidi B **215**, 199 (1999).
[14] T. Ikawa and K. Cho, Phys. Rev. B **66**, 085338 (2002).
[15] E. L. Ivchenko, M. M. Voronov, M. V. Erementchouk, L. I. Deych, and A. A. Lisyansky, Phys. Rev. B **70**, 195106 (2004).
[16] L. Pilozzi, A. D'Andrea, and K. Cho, Phys. Rev. B **69**, 205311 (2004).
[17] N. Tomassini, D. Schiumarini, L. Pilozzi, and A. D'Andrea, Phys. Rev. B **75**, 085317 (2007).
[18] C. Janot, *Quasicrystals: A Primer* (Clarendon, Oxford, UK, 1994).
[19] E. L. Ivchenko, *Optical Spectroscopy of Semiconductor Nanostructures* (Alpha Science International, Harrow, UK, 2005).